School of Economics and Management – University of Aarhus

Economic Modelling and Simulations - Term Structure Dynamics and Supercomputers

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Outline for the talk

- ⋆ A brief introduction of us.
- ⋆ The objective of our research project
- ⋆ Our working hypothesis to explain the term structure dynamics
- ⋆ New computer intensive methods for estimating our models
- ⋆ Results
- ⋆ Other applications of supercomputers in economics and finance

Introduction

- ⋆ Bent Jesper Christensen: 25 nodes (two dual core) in the Grendel Cluster, DCSC-AAU
- ⋆ Software: Fortran and MPI
- ⋆ Excellent support from Niels Carl
- ⋆ We are "plain economics" and have no special background in computer science.

Objective of the research project

- ⋆ Explain the link between the term structure of interest rates and the macro economy
- ⋆ Improve existing estimation methods for structural estimation
  - – Kalman Filtering in a non-linear and non-normal state space systems
  - – Evolutionary optimization algorithm suitable for multiprocessing

Economic Intuition

⋆ The starting point: Central Banks determine the short interest rate - stabilizing GDP and inflation

$\Longrightarrow$ Central Banks also affect the medium and long end of the term structure of interest rates

⋆ But, the macro-finance link: often not present in term structure models or in macro economy models!

⋆ The potential gain of getting this link right:

1. understand *why* interest rates were high in the 1970's and low in 1990's?
2. financial markets contain much information
   (a) interest rates may improve forecasts of GDP, consumption
   (b) interest rates may help the Central Bank to stabilize GDP and inflation

Economic Intuition

- ⋆ Currently, there is no model capable of explaining all the interest rates *and* explaining the macro economy from first principles (Diebold, Piazzesi and Rudebusch (2005))
- ⋆ Hence, the "link" between the macro economy and the term structure is missing

Our working hypothesis

- ⋆ Why are the existing macro models not able to explain the all the interest rates?
- ⋆ Theoretically (Cochrane, 2001):
  $r_{t,k} = E_t \left[f \left(c_t, c_{t+1}, ...c_{t+k}, \pi_{t+1}, ...\pi_{t+k} \right) \right]$
  c_t=consumption at time t
  π_t=inflation rate at time t
- ⋆ Actual consumption data has a trend - but this is ignored in the models
- ⋆ Our working hypothesis: introducing a consumption trend into macro models will enable these models to explain the term structure

Procedure to test the hypothesis

⋆ A test of this hypothesis requires solving the following three problems

1. Bounding the utility function when consumption has a trend
2. Improving existing methods for evaluating likelihood functions for DSGE models
3. Developing methods to optimize likelihood functions for DSGE models

The first problem

- $\star$ Macro economic models are constructed based on an assumption of optimizing agents
- $\star$ For instance, the households behaviour is derived from

$$\underset{c_t}{Max} \quad U_t = \sum_{l=0}^{\infty} \beta^l E_t \left[u \left(c_{t+l} \right) \right]$$

$s.t.$ budget constraints and $\beta < 1$

- $\star$ But, a trend in consumption may imply that $U_t = \infty$
- $\star$ I derive conditions on β which ensure that $U_t < \infty$.
- $\star$ Thus it is possible to have a trend in consumption in these models

The second problem

- ⋆ The economy is described by a set of parameters. For instance:
  1. The households preferences
  2. The reaction of the Central Bank to inflation
  3. etc.
- ⋆ These parameters are determined by maximizing the probability that our model has generated the actual data
- ⋆ Calculating this probability is very difficult in our case

$$\mathbf{y}_t = \mathbf{g}\left(\mathbf{x}_t, \boldsymbol{\epsilon}_t\right)$$
$$\mathbf{x}_{t+1} = \mathbf{f}\left(\mathbf{x}_t, \mathbf{v}_t\right)$$

$\mathbf{y}_t =$contains all interest rates and macro variables at time t
$\mathbf{x}_t =$unobserved regressors at time t

The second problem

⋆ Hence, we need to do non-linear regression without observing the regressors!

⋆ In the linear and normal case, the Kalman Filter can be applied

⋆ In the non-linear and non-normal case, Sequential Monte Carlo Methods can be applied

  Posterior state distributions are approximated by a sequence of:

  – Importance sampling
  – Resampling

⋆ Existing methods are very time consuming and not too precise

⋆ We modify these methods such that they are less time consuming and more robust

The third problem

- ⋆ To find the parameters in our economy we need to optimize the likelihood function

$$\underset{\mathbf{x}}{Max} \quad L(\mathbf{x})$$

- ⋆ Very difficult for these economies and existing methods performe very poorly
- ⋆ We modify an evolutionary optimization routine by Hansen et al (2003) and we show that this modified routine is quite successful at optimize likelihood functions for these economies

The third problem

- For each iteration a set of λ points in the search space is found by sampling from a n-dimensional multivariate normal distribution.

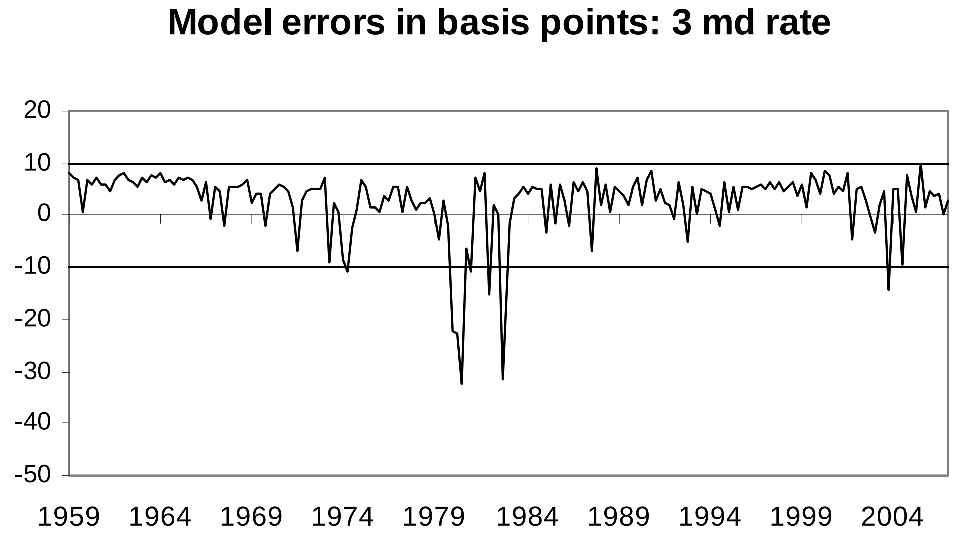
$$\mathbf{x}_i^{(g+1)} \sim N\left(\langle \mathbf{x} \rangle_w^{(g)}, {\sigma^{(g)}}^2 \mathbf{C}^{(g)}\right) \quad i = 1, ..., \lambda$$

- The lower and upper bound for the search space: $a(h) \leq x(h) \leq b(h)$ for all h.
- If $\mathbf{x}_i^{(g+1)}$ violates these bounds or any of the other constraints - re-sample until a value of $\mathbf{x}_i^{(g+1)}$ is found where the objective function is defined.
- Based on the λ points in generation $g+1$ the mean and the covariance matrix are updated to improve the search in the next iteration where generation $g+2$ is created in a similar manner.
- Multiprocessing is easy in this framework: Master vs. Slaves

So, does the working hypotesis hold?

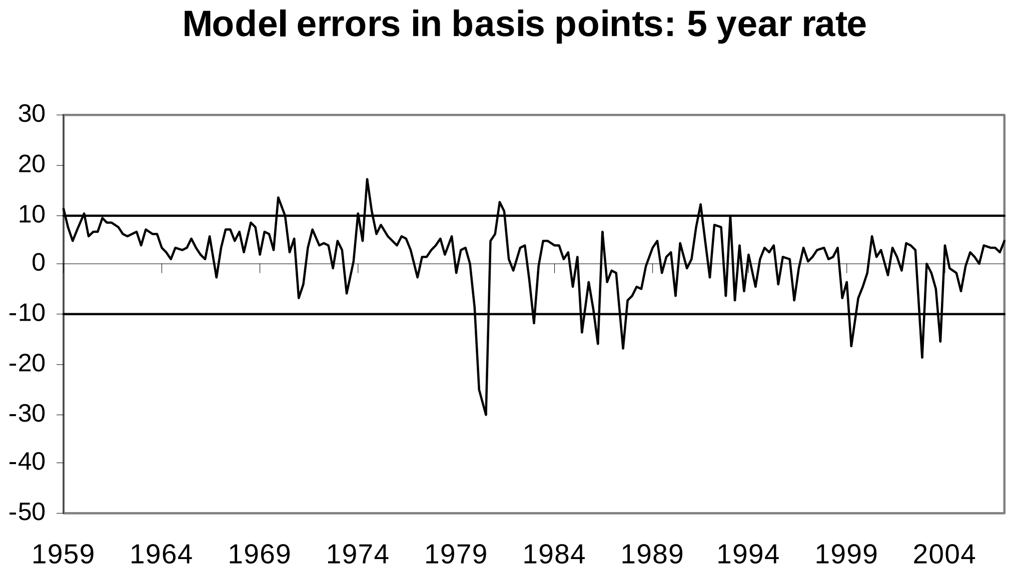
- ⋆ Estimation of the model is done on post-war US data.
- ⋆ 30-35 parameters are estimated by Maximum Likelihood methods
- ⋆ Multiprocessing is implemented in the optimization step of this estimation routine.
- ⋆ Results:

So, does the working hypotesis hold?



$$Model\ Error_t = r_{t,3}^{\text{actual}} - r_{t,3}^{\text{model}}$$

So, does the working hypotesis hold?



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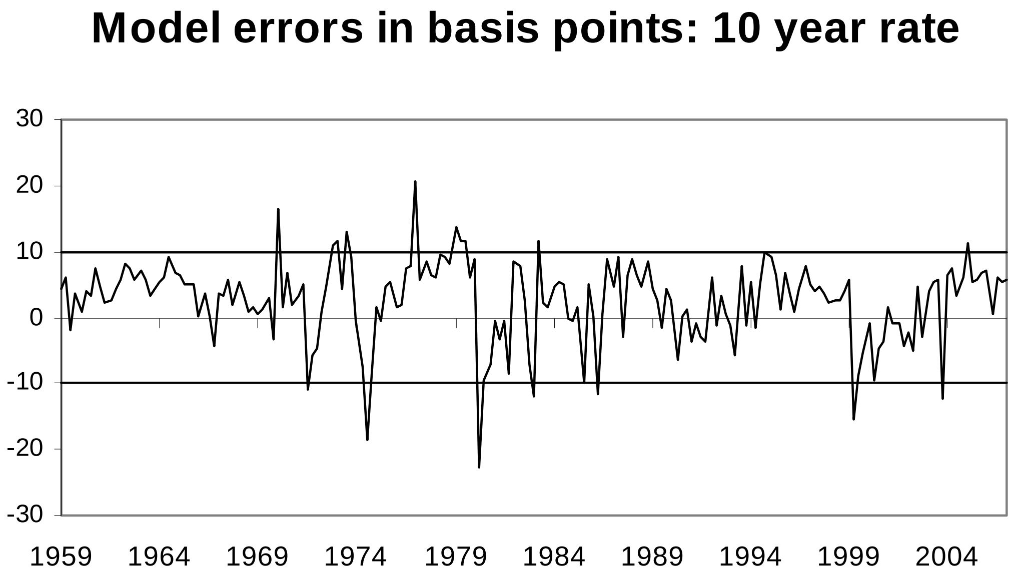

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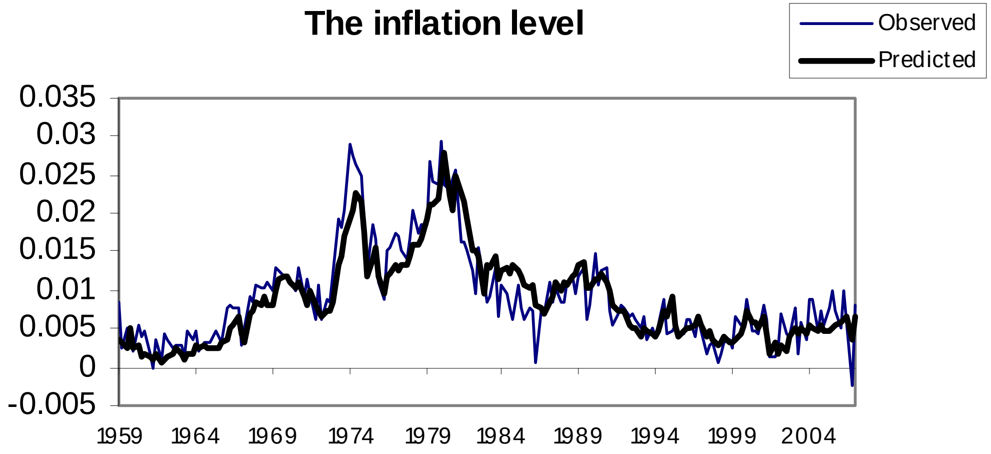

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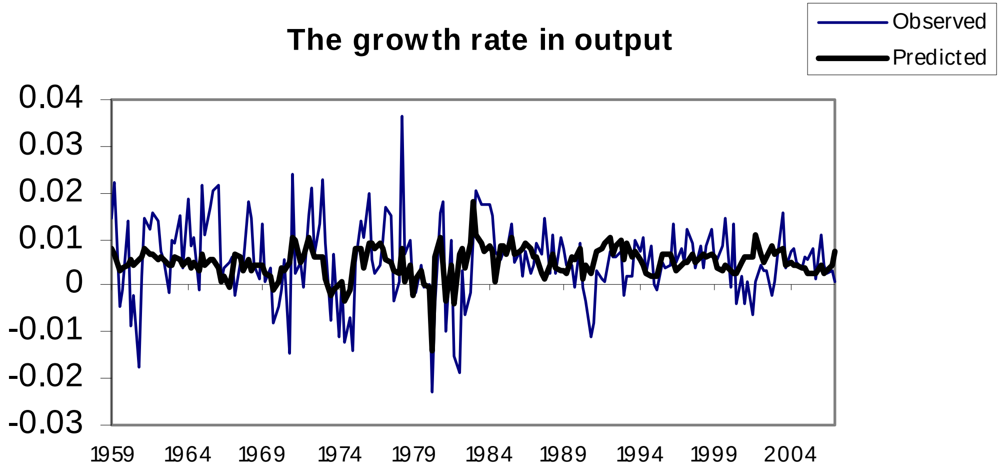

Computing in economics and finance

- ⋆ Economic model of market, or economy, or problem at hand
- ⋆ Economic theory delivers model
- ⋆ Language of mathematics and stochastics
- ⋆ Model assumes list of agents, their preferences, and endowments
- ⋆ Also assumes the rules of the game—how agents interact, e.g., trade
- ⋆ Sometimes model can be solved analytically—equilibrium found
- ⋆ Equilibrium (John Nash): Each agent optimizes, given others' actions
- ⋆ Competitive equilibrium: Price takers, can ignore effect of own trade on prices

Operationalizing the model

- ⋆ To apply model, e.g., for policy purposes, must estimate unknown parameters
- ⋆ Parameters of preferences, risk aversion, discounting, production functions, stochastic shocks
- ⋆ Given data on economic outcomes and model, estimate parameters
- ⋆ Statistical models, many parameters, many observations
- ⋆ Heavy computational burden

More complications

- ⋆ Frequently, model cannot be solved analytically
- ⋆ Even without data, and for assumed parameters, model must be solved numerically
- ⋆ Combination: Estimation of numerically solved model
- ⋆ Iterative procedure: At each trial parameter vector, resolve model
- ⋆ Typical model solutions: (Nested) fixed points, PDEs
- ⋆ Typical estimation procedures: Maximum likelihood, method of moments

Special issues

* ⋆ Model yields likelihood outside exponential family, no sufficient statistics—sample size does increase burden
* ⋆ No analytical likelihood or moments, must simulate
* ⋆ Must resample (jackknife, bootstrap) to get sampling distribution
* ⋆ Model error: Modelling thinking and reacting individuals
* ⋆ Imposing rationality (not pure description) adds computational complexity and usefulness
* ⋆ Nonexperimental data: Market place, economy

What parallelizes?

- ⋆ Likelihood for cross-section, not time series
- ⋆ Simulation/PDE for likelihood/moments when not known analytically
- ⋆ Resampling schemes
- ⋆ Global (annealing, not gradient based) optimization of likelihood/moments objective
- ⋆ Optimization given actions of others: Some game solutions (communication between iterations)

Danish labor market data

- ⋆ Best in world! Can see all employees in all firms
- ⋆ Wage, job function, demographics, job moves
- ⋆ Accounting information, productivity
- ⋆ All anonymous/cryptized, through Statistics Denmark
- ⋆ Researcher access only by application, micro data may not be moved from Statistics Denmark servers—a confidentiality issue
- ⋆ No similar register data base access in other countries
- ⋆ Computers must be hosted and housed by Statistics Denmark
- ⋆ Users and system manager connect from own workplace through web based secure gateway
- ⋆ Former Uni•C Danish Supercomputing Facility (DTU) grant: Estimation of dynamic programming model of married couples' joint retirement decisions

Worker reallocation across firms

- ⋆ B.J. Christensen, R. Lentz, D.T. Mortensen, G.R. Neumann, and A. Wervatz: On the Job Search and the Wage Distribution. *Journal of Labor Economics,* 2005
- ⋆ Lentz Danish (at Wisconsin)
- ⋆ Mortensen American (Niels Bohr Professor, AU, Danish National Research Foundation)
- ⋆ Workers search while employed for better jobs, reallocate for wage improvements
- ⋆ Nash equilibrium in firms' wage posting strategies
- ⋆ Higher wage means lower profit per worker, but hire faster, keep workers longer, larger labor force
- ⋆ Explains all of wage increase over time without skill improvement
- ⋆ Uses the unique Danish matched employer-employee data

Explaining economic growth through worker reallocation

- ⋆ R. Lentz and D.T. Mortensen: Labor Market Frictions, Firm Heterogeneity and Aggregate Employment and Productivity
- ⋆ Currently applying for new cluster consisting of 20 standard nodes, each with 2 dual-core CPUs, 8 GB memory, and 1 file server
- ⋆ Acquisition, installations, maintenance to be handled by CSCAA (Niels E. Christensen, Niels Carl Hansen)
- ⋆ Model extends 'An Empirical Model of Growth through Innovation,' *Econometrica*, forthcoming
- ⋆ This explains stylized facts: Large productivity dispersion across firms; persistent productivity ranks of firms; and large fraction of aggregate productivity growth is consequence of reallocation of workers and complementary input from less to more productive production units
- ⋆ Growth determined by choices made by the firm regarding production innovations

Explaining economic growth through worker reallocation

- ⋆ More innovation increases future growth, value added, total wages, labor force size of the firm
- ⋆ Estimation by indirect inference: Simulated moments method
- ⋆ Empirical moments estimated in data
- ⋆ Model is solved and simulated for given parameter values, and the simulated data used to estimate moments
- ⋆ Change parameters until simulated moments match empirical
- ⋆ New model explaining aggregate employment and productivity will be solved and estimated by similar method

Current model

- ⋆ Has been solved at SSCC, Social Science Computing Cooperative, Wisconsin
- ⋆ The CPU consuming part of estimation is the data simulation and search for a minimum of an object function of a 16-dimensional parameter vector
- ⋆ The object function is non-smooth
- ⋆ Growth/death pattern for each firm is simulated for 7 years with a week as time step
- ⋆ Gives longitudinal data set for 5,000 firms for value added, wage sum, and labor force size
- ⋆ Simulations repeated 1000 times
- ⋆ Programs written in Fortran, run on SSCC cluster. Each CPU performs 20 simulations. Takes approx. 20,000 iterations to determine global minimum of objective function.

Current model

- ⋆ Bootstrap based on 500 repetitions used to compute confidence intervals
- ⋆ Approx. $\frac{1}{2}$ hour to find point estimate, 250 hours for confidence intervals
- ⋆ Due to different technologies, model is estimated on 3 different sectors: Manufacturing, construction, whole and retail
- ⋆ New project maybe carried out in Denmark

Other finance examples

- ⋆ Finance is the branch of economics concerned with stock prices, bond prices, interest rates, exchange rates, financial options, firm policies relating to debt, dividend payout, etc.
- ⋆ Examples of economics Nobel prizes to finance: Role of debt for firm value; pricing of financial option as function of underlying asset
- ⋆ Assume stochastic process for price of underlying asset, by no arbitrage get PDE for option price
- ⋆ Solve, e.g., by finite difference scheme, or simulation (Feynman-Kac)
- ⋆ High-frequency financial data: 5-minute returns, or transactions (trade and quote) data, up to multiple trades each second
- ⋆ Derivative (e.g., option) prices depend heavily on volatility of return
- ⋆ Volatility strongly persistent, long memory or fractionally integrated process (Mandelbrot, Hurst, Taqqu, Granger)

Research contributions

- ⋆ Volatility could be estimated *either* from high-frequency returns (realized volatility) on stock, bonds, or foreign exchange
- ⋆ *or* from option prices (implied volatility matching PDE solution and data):
- ⋆ B.J. Christensen and N.R. Prabhala: The Relation Between Implied and Realized Volatility. *Journal of Financial Economics*, 1998
- ⋆ Asymptotics should account for long memory or fractional integration of each series:
- ⋆ B.J. Christensen and M. Nielsen: Asymptotic Normality of Narrow-Band Least Squares in the Stationary Fractional Cointegration Model and Volatility Forecasting. *Journal of Econometrics*, 2006

Research contributions

- ⋆ How do the received lessons help us set the fair price of common stock?
- ⋆ B.J. Christensen and M. Nielsen: The Effect of Long Memory in Volatility on Stock Market Fluctuations. *Review of Economics and Statistics*, 2007
- ⋆ Doing the similar for interest rates: Is the current shape of the yield curve a crystal ball for future interest rates?
- ⋆ T. Björk and B.J. Christensen: Interest Rate Dynamics and Consistent Forward Rate Curves. *Mathematical Finance*, 1999. Reprinted in *The New Interest Rate Models*, ed. L. Hughston. London: Risk Books, 2000

Conclusion

- ⋆ Economics and finance: Have mathematical models and data
- ⋆ Must inform decision makers (politicians, central bankers, firm managers, ...)
- ⋆ Models must hold even under the counterfactual (policy experiments) and out-of-sample (forecasting)
- ⋆ People (economic agents) make rational forecasts when chosing actions
- ⋆ Numerical solution of model at each trial parameter vector
- ⋆ Estimation of unknown parameter vector
- ⋆ PDE, simulation, resampling
- ⋆ Some tasks parallelize, some don't
- ⋆ Some unique Danish advantages